Flow Profile Induced in Spinneret During Hollow Fiber Membrane Spinning

S. J. SHILTON

Department of Chemical and Process Engineering, University of Strathclyde, James Weir Building, 75 Montrose Street, Glasgow G1 1XJ, Scotland

Received 25 October 1996; accepted 12 November 1996

ABSTRACT: A methodology is presented to establish the flow profile induced in a spinneret during the spinning of hollow fiber membranes. The flow equations are derived for a power law fluid passing through a concentric annulus. The pressure drop, the velocity profile, the shear stress profile, and the shear rate profile induced during spinning can then be determined. This type of rheological knowledge is useful if membrane structure and properties are to be related to the flow conditions experienced in the spinneret. © 1997 John Wiley & Sons, Inc. J Appl Polym Sci **65:** 1359–1362, 1997

Key words: hollow fiber spinning; rheology; flow profile

INTRODUCTION

The rheological conditions established in the spinneret during hollow fiber membrane spinning are essentially those of a power law fluid flowing through a concentric annulus in axial flow. Hanks and Larsen[1] presented a nonempirical solution to this problem valid for all values of the power law index (not just for reciprocal integers, as in previous cases). However, they, and others[2] focused on the relationship between pressure drop and flow rate through the annulus. In this paper, the solution to the flow equations gives the velocity, shear stress, and shear rate profiles across the annulus during flow. This type of rheological knowledge is valuable if membrane structure and properties are to be related to the deformation history experienced in the spinneret.

Flow conditions during extrusion are known to affect the properties of textile fibers[3,4] and hollow fiber membranes[5–7] by altering molecular orientation. Membrane researchers now recognize that detailed structural information at the molecular level is required if the performance of membranes is to be more fully understood.[8–10] Workers are currently employing spectroscopic techniques to probe the active layer of the membrane and examine molecular orientation.[11–13] A recent publication[14] dealing with melt extrusion has clearly demonstrated that particles align along the line of shear and that this orientation is most marked at the wall of the filament where shear rates are highest during extrusion. In order to begin to address how anisotropy is induced in polymer molecules during hollow fiber membrane spinning, the flow conditions in the spinneret should be known. It is hoped that this paper will be regarded as a timely how-to guide for workers carrying out fundamental research in hollow fiber membrane spinning.

FLOW EQUATIONS FOR A CONCENTRIC ANNULUS

The flow in a spinneret during hollow fiber spinning is normally laminar: a Reynolds number of 0.5 is typical. All of the material presented in this paper is based on laminar flow. A force balance over an element in a circular conduit yields the following equation relating shear stress as a function of radius to pressure drop:

$$T = \frac{r}{2}\left(\frac{dP}{dZ} - \rho g\right) + \frac{K_1}{r} \tag{1}$$

This is a general expression from which flow conditions for particular circumstances are derived. For a plain circular pipe, the following boundary conditions prevail:

$$T = 0 \quad \text{at } r = 0, \quad \text{thus } K_1 = 0$$
$$U_Z = 0 \quad \text{at } r = R$$

and flow conditions can be analytically derived for both Newtonian, $T = \eta(dU_Z/dr)$, and power law, $T = K(dU_Z/dr)^n$, fluids. In the case of a concentric annulus (i.e., a spinneret), the boundary conditions are more complex: the radius at which $T = 0$ is not obvious and $K_1 \neq 0$

$$U_Z = 0 \quad \text{at } r = R_1$$
$$U_Z = 0 \quad \text{at } r = R_2$$

and flow conditions can be analytically derived only for Newtonian fluids.

Newtonian Fluids

The following equations are analytically derived for Newtonian flow through a concentric annulus using eq. (1) and the appropriate boundary conditions:

$$\frac{dP}{dZ} = \left(\frac{8Q\eta}{\pi}\frac{1}{\dfrac{(R_2^2 - R_1^2)^2}{\ln\left(\dfrac{R_2}{R_1}\right)} - (R_2^4 - R_1^4)}\right) + \rho g \tag{2}$$

$$U_Z = \frac{2Q}{\pi}\frac{\dfrac{(R_2^2 - R_1^2)\ln\left(\dfrac{R_2}{r}\right)}{\ln\left(\dfrac{R_2}{R_1}\right)} - (R_2^2 - r^2)}{\dfrac{(R_2^2 - R_1^2)^2}{\ln\left(\dfrac{R_2}{R_1}\right)} - (R_2^4 - R_1^4)} \tag{3}$$

$$\dot{\gamma} = \left(\frac{8Q}{\pi}\frac{1}{\dfrac{(R_2^2 - R_1^2)^2}{\ln\left(\dfrac{R_2}{R_1}\right)} - (R_2^4 - R_1^4)}\right) \times \left(\frac{r}{2} + \frac{(R_2^2 - R_1^2)}{4r\ln\left(\dfrac{R_1}{R_2}\right)}\right) \tag{4}$$

$$T = \eta\dot{\gamma} \tag{5}$$

Power Law Fluids (Flow Equations and Solution Methodology)

The flow equations relating to a power law fluid through an annulus cannot be derived analytically because of the integration difficulties with the power law index n. The solution to the problem involves numerical integration and is iterative.

The general flow equation [eq. (1)] and boundary conditions have been given. The constant K_1 can be represented by defining r_0 as the radius at which $T = 0$, thus

$$K_1 = -\frac{r_0^2}{2}\left(\frac{dP}{dZ} - \rho g\right)$$

and

$$T = \frac{1}{2}\left(\frac{dP}{dZ} - \rho g\right)\left(r - \frac{r_0^2}{r}\right) \tag{6}$$

For a power law fluid,

$$T = K\dot{\gamma}^n = K\left(\frac{dU_Z}{dr}\right)^n \tag{7}$$

Combining eqs. (6) and (7),

$$dU_Z = \left(\frac{1}{2K}\left\{\frac{dP}{dZ} - \rho g\right\}\left\{r - \frac{r_0^2}{r}\right\}\right)^{1/n} dr \tag{8}$$

Normalizing eq. (8) with respect to R_2 and collecting constants,

$$dU_Z = K_2\left(\frac{\lambda^2}{\xi} - \xi\right)^{1/n} d\xi \qquad (9)$$

where $\xi = r/R_2$, $\lambda = r_0/R_2$, and

$$K_2 = R_2\left(\frac{R_2}{2K}\left\{\rho g - \frac{dP}{dZ}\right\}\right)^{1/n} \qquad (10)$$

From eq. (9), defining the standardized velocity

$$u_Z = U_Z/K_2 \qquad (11)$$

defining the aspect ratio of the annulus $\sigma = R_1/R_2$, and using the boundary condition $u_Z = 0$ at $\xi = \sigma$,

$$u_{Z|_\xi} = \int_\sigma^\xi \left(\frac{\lambda^2}{\xi} - \xi\right)^{1/n} d\xi \quad \sigma \le \xi \le \lambda \quad (12)$$

Similarly, using the boundary condition $u_Z = 0$ at $\xi = 1$

$$u_{Z|_\xi} = \int_\xi^1 \left(\xi - \frac{\lambda^2}{\xi}\right)^{1/n} d\xi \quad \lambda \le \xi \le 1 \quad (13)$$

Combining eqs. (12) and (13) gives the following expression, which is solved numerically for λ by trial and error:

$$\int_\sigma^\lambda \left(\frac{\lambda^2}{\xi} - \xi\right)^{1/n} d\xi - \int_\lambda^1 \left(\xi - \frac{\lambda^2}{\xi}\right)^{1/n} d\xi = 0 \quad (14)$$

Evaluating λ, which represents the radius at which $T = 0$, is the crux of the problem. Once λ is found [eq. (14)], the flow conditions are easily determined. The standardized velocity profile in the annulus is evaluated numerically from eqs. (12) and (13). Using the velocity profile data the standardized volumetric flowrate, q, can in turn be numerically determined since

$$q = \int_\sigma^1 \xi u_{Z|_\xi}\, d\xi \qquad (15)$$

The actual volumetric flowrate through the annulus is given by

$$Q = 2\pi R_2^2 K_2 q \qquad (16)$$

K_2 can be evaluated for any particular volumetric flowrate from eq. (16). The pressure drop through the annulus is then calculated from eq. (10). The actual velocity profile is determined from eq. (11) and the shear stress profile from eq. (6). Finally, the shear rate profile is evaluated from eq. (7).

ESTABLISHING THE FLOW PROFILE IN PRACTICE

A preliminary measurement of spinning dope viscosity should be made (e.g., zero shear viscosity) using a rheometer. Equation (4) can then be used to give an initial estimation of the shear rates occurring during spinning. (The equation will show that shear rates in excess of 10,000 s^{-1} are often experienced at the walls of the spinneret.) The full flow curve for the dope (the relationship between shear stress and shear rate) can then be established over this shear rate range. The rheological measurements will probably show that the spinning dope behaves as a power law fluid. The power law index n and constant K are determined from the rheological data.

In accordance with the methodology described in the previous section, the power law flow equations are then solved for n and K, and the particular spinning conditions of interest, R_1, R_2, Z, ρ and Q, to yield the pressure drop, the velocity profile, the shear stress profile, and the shear rate profile across the annulus during spinning.

The author has written a computer program to carry out this procedure. A typical data file produced by the program is shown in Table I. A copy of the utility can be obtained by writing to the author.

NOMENCLATURE

ξ	normalized radius
g	acceleration due to gravity
$\dot{\gamma}$	shear rate
K	constant
η	viscosity
n	constant
ρ	density
P	pressure
Q	flowrate
q	standardized flowrate
r	radius
r_0	radius at which shear stress is equal to zero
R	radius of pipe
R_1	inner radius of spinneret

Table I Flow Conditions in Spinneret

Radius (mm)	Velocity (cm/s)	Shear Stress (N/m^2)	Shear Rate (s^{-1})	Radius (mm)	Velocity (cm/s)	Shear Stress (N/m^2)	Shear Rate (s^{-1})
0.1650	0.0000	116276	10976	0.2352	23.2012	11271	202
0.1677	2.8331	110241	10019	0.2379	23.1285	15316	341
0.1704	5.4152	104330	9117	0.2406	23.0145	19318	508
0.1731	7.7609	98538	8267	0.2433	22.8522	23278	699
0.1758	9.8841	92860	7468	0.2460	22.6352	27196	912
0.1785	11.7981	87290	6718	0.2487	22.3578	31075	1146
0.1812	13.5158	81823	6014	0.2514	22.0147	34916	1399
0.1839	15.0495	76455	5354	0.2541	21.6007	38720	1670
0.1866	16.4108	71182	4737	0.2568	21.1113	42488	1958
0.1893	17.6113	66000	4162	0.2595	20.5420	46222	2262
0.1920	18.6620	60904	3627	0.2622	19.8887	49921	2580
0.1947	19.5734	55891	3131	0.2649	19.1473	53588	2913
0.1974	20.3561	50958	2673	0.2676	18.3143	57224	3260
0.2001	21.0201	46102	2252	0.2703	17.3858	60828	3619
0.2028	21.5752	41319	1867	0.2730	16.3587	64403	3991
0.2055	22.0312	36606	1517	0.2757	15.2295	67949	4375
0.2082	22.3976	31961	1202	0.2784	13.9953	71467	4770
0.2109	22.6837	27382	923	0.2811	12.6529	74958	5176
0.2136	22.8989	22865	678	0.2838	11.1995	78422	5592
0.2163	23.0528	18409	468	0.2865	9.6324	81860	6018
0.2190	23.1546	14010	293	0.2892	7.9488	85273	6454
0.2217	23.2142	9668	155	0.2919	6.1463	88663	6900
0.2244	23.2419	5380	57	0.2946	4.2222	92028	7354
0.2271	23.2490	1144	4	0.2973	2.1742	95370	7817
0.2298	23.2549	3042	21	0.3000	0.0000	98690	8289
0.2325	23.2403	7180	93				

Dope: 40% w/w polysulfone in dimethylformamide; inner radius of annulus, .165 (mm); outer radius of annulus, .3 (mm); spinneret length, 1.25 (mm); power law index, .584 (—); power law constant, 508 (based on SI units); dope density, 1 (g/cm^3); dope extrusion rate, 2 (cm^3/min); radius at zero stress, 0.2278 (mm); maximum velocity, 23.2491 (cm/s); pressure drop across spinneret, 19.4314 (bar).

R_2 outer radius of spinneret
σ spinneret aspect ratio
T shear stress
U_Z velocity
u_Z standardized velocity
λ normalized radius at which shear stress is equal to zero
Z distance

REFERENCES

1. R. W. Hanks and K. M. Larsen, *Ind. Eng. Chem. Fundam.,* **18,** 33 (1979).
2. N. Prasanth and U. V. Shenoy, *J. Appl. Polym. Sci.,* **46,** 1189 (1992).
3. D. R. Paul, *J. Appl. Polym. Sci.,* **13,** 817 (1969).
4. A. Ziabicki, *Fundamentals of Fibre Formation,* Wiley, New York, 1976.
5. O. M. Ekiner and G. Vassilatos, *J. Membr. Sci.,* **53,** 259 (1990).
6. P. Aptel, N. Abidine, F. Ivaldi, and J. P. Lafaille, *J. Membr. Sci.,* **22,** 199 (1985).
7. S. J. Shilton, G. Bell, and J. Ferguson, *Polymer,* **35,** 5327 (1994).
8. I. Pinnau, M. W. Hellums, and W. J. Koros, *Polymer,* **32,** 2612 (1991).
9. P. H. Pfromm, I. Pinnau, and W. J. Koros, *J. Appl. Polym. Sci.,* **48,** 2161 (1993).
10. S. J. Shilton, G. Bell, and J. Ferguson, *Polymer,* **37,** 485 (1996).
11. K. C. Khulbe, S. Gagné, A. Tabe Mohammadi, T. Matsuura, and A.-M. Lamarche, *J. Membr. Sci.,* **98,** 201 (1995).
12. S. J. Shilton, A. F. Ismail, P. J. Gough, I. R. Dunkin, and S. L. Gallivan, *Polymer,* **38,** 2215 (1997).
13. A. F. Ismail, S. J. Shilton, I. R. Dunkin, and S. L. Gallivan, *J. Membr. Sci.,* **126,** 133 (1997).
14. M. Bousmina and R. Muller, *Rheol. Acta,* **35,** 369 (1996).